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An Atemporal Microeconomic Theory and an Empirical Test of Price-Induced Technical Progress

by

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Abstract

An exhaustive comparative statics analysis of a general price taking cost-minimizing model of the firm operating under the influence of price-induced technical progress is carried out from a dual vista. The resulting refutable implications are observable and thus amenable to empirical verification, and take on the form of a symmetric and negative semidefinite matrix. Using data from individual cotton gins in California's San Joaquin Valley, we empirically test the complete set of implications of the price-induced technical progress theory using both classical and Bayesian statistical procedures. We find that the data are fully consistent with the atemporal, cost-minimizing, price-induced microeconomic theory of technical progress.

Keywords: Price-induced technical progress, Comparative statics
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1. Introduction

The hypothesis that changes in the relative prices of factors of production influence the magnitude and direction of technical progress (TP) is of old vintage, most probably due to Hicks (1932, p. 124). This conjecture implies that relative factor prices serve a dual function, to wit, the prototypical role as signals of resource scarcity, and the novel role as determinants of the firm's technology choice. The premise is that a firm is confronted with an array of feasible production techniques, and the relative prices of the factors of production influence the choice of technique. Given a choice of technique, the firm then goes about its customary optimization process by choosing its factors of production, subject to the chosen technology. Hayami and Ruttan (1971) revitalized Hicks' conjecture and made important contributions to the explanation of the magnitude and direction of TP in the American and Japanese agricultural sectors using the relative price hypothesis. They provided some empirical evidence in support of this conjecture but their studies did not culminate in a rigorous statistical test of the hypothesis.

Technical progress can be fruitfully classified into the two broad phases of innovation and adoption. In general, firms self-select into either one or the other phase, rarely into both. Firms that self-select principally into the innovation phase have the explicit objective of discovering new techniques of production and/or consumption. Examples of this class are the biotechnology and pharmaceutical firms. Their R&D budget constitutes the principal means for pursuing the innovation objective. Small firms that are tendentially price-takers, on the other hand, typically self-select into the adoption phase of TP. An important example of such entities is provided by agricultural firms. Their path toward TP is mainly characterized by the adoption of already available and marketable techniques of production and consumption as motivated by their profit objective. The skill of these entrepreneurs together with the market conditions as perceived by them allow an adoption rate of new techniques that, a posteriori, is interpreted and measured as a rate of TP. For these firms, the principal determinants of their TP are the plant size and equipment decisions, together with the relative prices of inputs and outputs under different technical possibilities.

Furthermore, the analysis of TP can be performed either at the firm, industry, or economy level. Our paper focuses on the firm level in order to extract unambiguous and empirically verifiable relations that form the basis for an exhaustive and direct test of the price-induced TP hypothesis. A principal deficiency of many studies dealing with TP, in fact, is the dearth of empirically testable hypotheses that can be used to either support or invalidate a specified model. For example, the earlier work of Paris (1993) and Paris and Caputo (1995) on price-induced TP was only partly successful, in that although they derived the comparative statics properties of the transpose (or reciprocal) specification of the profit maximization problem, they did not derive the comparative statics properties of the profit maximization problem itself, which is the problem of direct economic interest. The recent work of Paris and Caputo (2001), in contrast, presented a complete comparative statics characterization of a price-taking profit-maximizing model of the firm operating under the influence of price-induced TP.

The majority of empirical studies about TP are static or atemporal in nature, even though they may be based on time-series information. While we do indeed recognize the dynamic nature of TP, and believe that an intertemporal theory is more plausible and intellectually satisfying, *a priori*, the present paper concentrates instead on the price-induced TP hypothesis in a static or atemporal context. We do so in order to highlight the essential aspects of our model and to solidify our own understanding of its basic theoretical implications. Furthermore, we believe this is a necessary first step before tackling the more complicated dynamic theory, in complete agreement with Occam's razor. Moreover, the availability of a unique sample of data dealing with cotton ginning cooperative firms in California has provided the opportunity for investigating the price-induced TP hypothesis with a close match between the atemporal theory and its empirical implementation. We thus leave the intertemporal extension for future research.

Another important characteristic of the data is that the cotton ginning firms are *a priori* expected to be cost-minimizing, rather than profit-maximizing, enterprises for reasons that will become clear in section 6 when we discuss the industry and tests of the theory in detail. Hence, our first objective of this paper is to extend the derivation of the comparative statics properties of the price-induced TP hypothesis to the case of a price-taking cost-minimizing firm. As in previous work [see Paris (1993) and Paris and Caputo (1995, 2001)], we incorporate relative factor prices (input prices normalized by the single output price) explicitly into the production function. In this manner we succinctly capture the role of relative prices as shift parameters of the technology frontier. The double role postulated for relative prices, however, destroys the traditional comparative statics relations of the competitive firm. In order to recover refutable and empirically verifiable hypotheses for the cost-minimizing competitive firm operating under price-induced TP, it is therefore necessary to consider a more complex framework. The major result of our theoretical analysis is a set of comparative statics relations that depend upon primal and dual functions, and come in the form of a symmetric and negative semidefinite matrix of observable, and hence estimable, terms. Consequently, the empirical implementation of the comparative statics conditions developed in this paper requires, in general, the concomitant measurement of the cost function *and* the production function. This novel feature is not present in the paradigmatic models of the firm and thus creates the scaffolding for a specific logical test of the price-induced TP theory together with an added complexity when carrying out empirical tests of it.

The second objective of our paper, therefore, is to statistically test the full set of refutable and qualitative properties implied by the price-induced TP theory derived in section 3. We use classical and Bayesian statistical methods to conduct the estimation of the model and carry out the hypothesis tests. Our data consists of a combined time-series/cross-section of annual observations on individual cooperative cotton gins located in the San Joaquin Valley of California. The statistical analysis yields strong evidence in favor of the cost-minimizing price-induced TP model, but little evidence in favor of the profit-maximizing price-induced TP model.

2. Literature Review

To date, only a handful of papers have estimated a microeconomic model of the firm under the hypothesis of price-induced TP. All of the papers that estimated a production function included output and/or input prices, or some function of them, directly in the production function. None of these papers, however, tested the refutable implications of the price-induced TP hypothesis, and as a result, their intent and focus contrasts sharply with ours. Moreover, it is important to recognize that our paper deals with *price-induced* TP rather than *induced* TP. The induced TP literature of the sixties and seventies postulated that TP is induced by R&D activities. In contrast, the hypothesis of price-induced TP is based upon profitability considerations. This explains the paucity of papers directly related to ours, whether theoretical or empirical. In passing, note that there is a substantial literature that has looked at the theoretical and empirical link between TP and environmental policy. Since this literature is tangential to our focus, we refer readers interested in such matters to the recent survey article by Jaffe *et. al*. (2002).

Fulginiti and Perrin (1993) used a combined time-series/cross-section sample of agricultural production and price data from 18 countries to estimate a variable coefficient Cobb-Douglas meta-production function. They posited that the coefficients are functions of the output price and a few input prices, and other pertinent technology changing variables, using five-year moving averages for the prices. Celikkol and Stefanou (1999) used annual data on the U.S. food processing and distribution sector to estimate a price dependent generalized Leontief production function. They included the current output and input prices as well as a three-year moving average of input prices, the latter designed to capture the role of prices as technology shifters. Departing from the use of aggregate data, Oude Lansink, Silva, and Stefanou (2000) estimated firm-specific and price dependent quadratic production frontiers for Dutch glasshouse firms using the generalized maximum entropy method. They included a three-year moving average of past energy prices as an argument of the production frontier to capture the role of prices as technology shifters. Finally, Peeters and Surry (2000) used times series data on the feed manufacturing industry in Belgium to estimate a multiple-output symmetric generalized McFadden cost function. They included lagged input prices as arguments of the cost function in order to capture the price-induced TP effect.

3. Refutable Propositions of the Price-Induced Technical Progress Hypothesis

In this section we formally develop the complete set of refutable and testable implications of a general price-taking cost-minimizing model of the firm operating under the influence of price-induced TP. Along the way, we point out the features of the model's qualitative properties that depart from their traditional counterparts. We formally model the Hicks' conjecture of price-induced TP by explicitly considering relative prices as determinants of the firm's production possibility set. Such a formulation implies that relative prices enter the production function and thus serve in the nontraditional role as shifters of the technology.

In order to gain some valuable insight into the plausible ways in which relative prices may influence the choice of techniques, it is useful to begin by quoting McFadden [Fuss and McFadden (1978, p. 6)]:

> The production possibility set of a firm is determined first by the state of technological knowledge and physical laws. … There may be further limitations on the availability of techniques due to imperfect information and legal restrictions (e.g. patent agreements, pollution control standards, safety standards). Non-transferable commodities, such as 'managerial capacity', climate, and environmental factors, may also enter in the determination of production possibilities. Finally, in most economic problems, the firm will be required to meet restrictions on some input and output quantities due to prior contracts, quotas, rationing, or 'hardening' of commodities following *ex ante* decisions. Common examples are commitments to fixed plant and equipment inputs, and contracts to purchase inputs (e.g. labor services) or supply outputs.

It is important to observe that contracts to purchase inputs and supply outputs imply knowledge of, and decisions based on, the corresponding input and output prices. Similarly, commitments to fixed plant and equipment inputs necessarily depend on input and output price expectations which, in the atemporal context of this paper, necessarily collapse to the current input and output prices.

The literature on TP also presents several statements concerning the role of relative prices in influencing the choice of production technique. For example, Arrow (1969, p. 29) wrote: "From studies of Griliches (1957) and Mansfield (1968, Part IV) we know that the diffusion of technological knowledge, at least within a given economy, is partly governed by profitability considerations." Griliches (1957, p. 519) has emphasized repeatedly the dependence of TP in the cultivation of hybrid corn upon profitability. One of his fifteen such references states: "…our results do suggest that a substantial proportion of the variation in the rate of acceptance of hybrid corn is explainable by differences in the profitability of the shift to hybrids in different parts of the country." Continuing the citation of established economists who advanced the conjecture that TP may depend on profitability considerations, Hirsch (1969, p. 38) stated: "And although the formal neoclassical models of the firm do not explicitly show the intertemporal trade-offs, the engineer-manager is assumed to choose the most profitable techniques of production from among all possible production functions." Profitability, of course, depends crucially on relative prices.

The aforementioned conjectures and empirical evidence relating TP to profits suggests that output and input prices enter the production function as shifters of the technology frontier. This is therefore how we intend to formally model the price-induced TP hypothesis of Hicks (1932). Our formal specification of priced-induced TP is thus perfectly analogous to the introduction of commodity prices into the direct utility function in order to account for Patinkin's conjecture about real cash balances [see Samuelson and Sato (1984) and Paris and Caputo (2002)]. With this background in mind, we now proceed more formally.

To begin, let $\mathbf{x} \in \Re_{++}^N$ be the vector of variable inputs used by the firm purchased at the market determined price vector $\mathbf{W} \in \Re_{++}^N$, and let $y \in \Re_{++}$ be the output of the single good produced by the firm which it sells at the market determined price $P \in \Re_{++}$. Define the *relative price vector* of the factors of production by $\mathbf{w} \overset{\text{def}}{=} \mathbf{W}/P \in \Re_{++}^N$. Note that we treat the vectors $\mathbf{x} \in \Re_{++}^N$, $\mathbf{W} \in \Re_{++}^N$, and $\mathbf{w} \overset{\text{def}}{=} \mathbf{W}/P \in \Re_{++}^N$ as column vectors throughout.

Given the profitability considerations in the choice of techniques, the technological and contractual environment of the firm depends on output and input prices $(P, \mathbf{W})$ as well as an index t representing exogenous technical change. As a result, the *production possibility set* of the firm may be defined as

$$Y(P,\mathbf{W},t) \overset{\text{def}}{=} \left\{ (y,\mathbf{x}) \in \Re_{++}^{N+1} \,\middle|\, (y,\mathbf{x}) \text{ is feasible in the environment } (P,\mathbf{W},t) \right\}. \qquad (1)$$

In light of definition (1), the firm's *input requirement set* is defined as

$$V(y,P,\mathbf{W},t) \stackrel{\text{def}}{=} \left\{\mathbf{x} \in \Re^N_{++} \,\middle|\, (y,\mathbf{x}) \in Y(P,\mathbf{W},t)\right\}. \tag{2}$$

The value of the *production function* $F(\cdot)$ is then defined as

$$F(\mathbf{x};P,\mathbf{W},t) \stackrel{\text{def}}{=} \max\left\{y \in \Re_{++} \,\middle|\, \mathbf{x} \in V(y,P,\mathbf{W},t)\right\}. \tag{3}$$

Note that the semicolon in the arguments of $F(\cdot)$ separates the vector of decision variables $\mathbf{x} \in \Re^N_{++}$ from the vector of exogenous variables $(P,\mathbf{W},t)$. We assume that $F(\cdot):\Re^{2N+2}_{++} \to \Re_{+}$ is $C^{(2)}$ on its domain. To capture the central role that relative prices play in determining the choice of technology under the price-induced TP hypothesis, as asserted by Hicks (1932), we assume that the production function $F(\cdot)$ is positively homogeneous of degree zero in the output price $P \in \Re_{++}$ and input price vector $\mathbf{W} \in \Re^N_{++}$, that is, $F(\mathbf{x};\theta P,\theta\mathbf{W},t) \equiv F(\mathbf{x};P,\mathbf{W},t)$ for all $\theta \in \Re_{++}$. Upon defining $\theta \stackrel{\text{def}}{=} P^{-1} \in \Re_{++}$, we may rewrite the above identity as $F(\mathbf{x};1,\mathbf{w},t) \equiv F(\mathbf{x};P,\mathbf{W},t)$, without loss of generality. Finally, defining $f(\mathbf{x};\mathbf{w},t) \stackrel{\text{def}}{=} F(\mathbf{x};1,\mathbf{w},t)$ yields the production function $f(\cdot):\Re^{2N+1}_{++} \to \Re_{+}$, which is defined in terms of relative prices, the form of economic interest and that which perfectly captures the Hicks conjecture.

The homogeneity assumption on the production function implies that a doubling of output and input prices does not change the firm's production function, which is what it means for relative, as opposed absolute, prices to influence the technology. This assumption also implies, via the first-order necessary conditions given by Eqs. (5) and (6) below, that the factor demand functions retain their desirable property of being homogeneous of degree zero in (P,W). Therefore, not assuming that $F(\cdot)$ is homogeneous of degree zero in (P,W) implies implausible economic behavior, to wit, a doubling of output and input prices leads to changes in the production function and in the factors of production employed. In others, by not assuming homogeneity, agents change their behavior even when no relative prices have changed. Note that our homogeneity assumption is wholly analogous to that made by Samuelson and Sato (1984) when commodity prices and money balances enter the direct utility function, namely, that the latter is homogeneous of degree zero in prices and money balances. Samuelson and Sato (1984) adopt this assumption so as to preserve the homogeneity of degree zero of the commodity demand functions in prices and income, and to capture the difference between money and goods.

It is important to remark that we have not made *a priori* assumptions about (i) the signs of the first-order partial derivatives of the production function with respect to the factors of production, (ii) the curvature of the production function with respect to the factors of production, (iii) the signs of the first-order partial derivatives of the production function with respect to the relative prices, or (iv) the curvature of the production function with respect to the relative prices. In spite of the lack of assumptions, we will demonstrate that empirically perceptible and refutable comparative statics properties are present in the model. The lack of assumptions about the production function therefore implies that the qualitative properties derived in Theorem 1 are fundamental to our price-induced TP theory. In other words, the refutable properties we establish in Theorem 1 are basic to the model since they are dependent only on the assumption of a unique interior solution of the optimization problem and the mathematical structure of it, and are not at all dependent on ad hoc assumptions or sufficient conditions imposed on the model.

Given these preliminaries, we may now state the price-taking cost-minimizing model of the firm operating under the influence of price-induced TP as

$$C(\mathbf{w},y,t) \overset{\text{def}}{=} \min_{\mathbf{x}} \left\{ \mathbf{w}'\mathbf{x} \ \text{ s.t. } \ y - f(\mathbf{x};\mathbf{w},t) \le 0 \right\}, \tag{4}$$

where the symbol "$'$" denotes transposition. Because of the generic nature of problem (4), we assume that it possesses a unique interior $C^{(1)}$ solution $\boldsymbol{\alpha} \mapsto \hat{\mathbf{x}}(\boldsymbol{\alpha})$ for all $\boldsymbol{\alpha} \in B(\boldsymbol{\alpha}^{\circ};\delta)$, where $B(\boldsymbol{\alpha}^{\circ};\delta)$ is an open $(N+2)$-ball centered at the point $\boldsymbol{\alpha}^{\circ} \in \Re_{++}^{N+2}$ with radius $\delta > 0$, and where $\boldsymbol{\alpha} \overset{\text{def}}{=} (\mathbf{w},y,t)$ is the given parameter vector of the problem. As a matter of notation, we adopt the conventions that (i) the derivative of a scalar-valued function with respect to a column vector is a row vector, and (ii) the order of a Hessian matrix of a scalar-valued function is given by (first subscript order)$\times$(second subscript order). Given this notational convention, the prototype model of the price-taking cost-minimizing model of the firm is a special case of problem (4), derived by setting $f_{\mathbf{w}}(\mathbf{x};\mathbf{w},t) \equiv \mathbf{0}'_N$, where $\mathbf{0}_N$ is the null column vector in $\Re^N$.

Defining $L(\mathbf{x},\lambda;\mathbf{w},y,t) \overset{\text{def}}{=} \mathbf{w}'\mathbf{x} + \lambda[y - f(\mathbf{x};\mathbf{w},t)]$ as the Lagrangian for problem (4), and assuming that the archetype nondegenerate constraint qualification holds at the solution, i.e., that $f_{x_n}\left(\hat{\mathbf{x}}(\mathbf{w},y,t);\mathbf{w},t\right) \neq 0$ for at least one value of the index n, the first-order necessary conditions are given by

$$L_{x_n}(\mathbf{x},\lambda;\mathbf{w},y,t) = w_n - \lambda f_{x_n}(\mathbf{x};\mathbf{w},t) = 0,\ n = 1,2,\ldots,N, \tag{5}$$

$$L_{\lambda}(\mathbf{x},\lambda;\mathbf{w},y,t) = y - f(\mathbf{x};\mathbf{w},t) \leq 0,\ \lambda \geq 0,\ L_{\lambda}(\mathbf{x},\lambda;\mathbf{w},y,t)\cdot\lambda = [y - f(\mathbf{x};\mathbf{w},t)]\cdot\lambda = 0. \tag{6}$$

Since $w_n > 0$, $n = 1,2,\ldots,N$, Eqs. (5) and (6) imply that the optimal value of the Lagrange multiplier is positive, i.e., $\hat{\lambda}(\mathbf{w},y,t) > 0$. In turn, this fact and Eq. (5) imply that the marginal product of each input is positive at the optimum, that is, $f_{x_n}\left(\hat{\mathbf{x}}(\mathbf{w},y,t);\mathbf{w},t\right) > 0$, $n = 1,2,\ldots,N$. These are *necessary* implications of problem (4), derived using only the assumption of an interior solution to the minimization problem (4). The property $f_{x_n}\left(\hat{\mathbf{x}}(\mathbf{w},y,t);\mathbf{w},t\right) > 0$, $n = 1,2,\ldots,N$, turns out to be important when we conduct the empirical tests of the price-induced TP theory in section 6. Moreover, it also proves that the aforementioned nondegenerate constraint qualification is satisfied for problem (4).

Several features of problem (4) deserve comment before presenting the central result of this section. First, the cost function $C(\cdot)$ defined in problem (4) is not concave in the relative factor prices, thereby implying that the archetypal comparative statics properties do not hold. This conclusion follows from the presence of the relative input prices in the production function and the fact that we did not make any assumptions about the curvature of the production function with respect to the relative input prices. Said differently, Eqs. (5) and (6) show that every relative input price appears in every first-order necessary condition because of the dependence of the production function on the relative factor prices. As a result, the standard comparative statics results no longer hold due to the appearance of principal minors that are not border preserving in the comparative statics expressions.

The second feature is the alteration of the form of the prototype Shephard's lemma. In particular, it is no longer true that the partial derivative of $C(\cdot)$ with respect to a relative input price equals the corresponding factor demand function under the cost-minimizing price-induced TP hypothesis. To see this, simply apply the envelope theorem to problem (4) to get

$$C_{w_n}(\mathbf{w},y,t) = \hat{x}_n(\mathbf{w},y,t) - \hat{\lambda}(\mathbf{w},y,t) f_{w_n}\left(\hat{\mathbf{x}}(\mathbf{w},y,t);\mathbf{w},t\right),\ n = 1,2,\ldots,N. \tag{7}$$

This perturbation of the prototype Shephard's lemma is yet another way to understand the generalization of the standard comparative statics results under the price-induced TP hypothesis.

The third feature is that $C(\cdot)$ is not necessarily increasing in the relative input prices. This conclusion follows from inspection of Eq. (7), the dependence of the production function on the relative input prices, and the absence of any assumptions on the signs of the first-order partial derivatives of the production function with respect to the relative input prices. Our empirical results, which will be discussed in section 6, show that the estimated cost function is decreasing in the relative price of labor, for example.

The fourth feature is that $C(\cdot)$ is not positively homogeneous of degree one in the relative input prices. This follows at once from the definition of $C(\cdot)$ given in Eq. (4), the dependence of the production function on the relative input prices, and the lack of any assumptions about the functional form of the production function with respect to the relative factor prices.

One property of the archetype cost function, however, does in fact carry over to $C(\cdot)$, namely, that it is increasing in output. This conclusion also follows directly from the envelope theorem and the aforementioned result $\hat{\lambda}(\mathbf{w},y,t) > 0$, since $C_y(\mathbf{w},y,t) = \hat{\lambda}(\mathbf{w},y,t)$. Using this envelope result, the modified Shephard's lemma in Eq. (7) can be rearranged to read

$$C_{w_n}(\mathbf{w},y,t) + C_y(\mathbf{w},y,t) f_{w_n}\left(\hat{\mathbf{x}}(\mathbf{w},y,t);\mathbf{w},t\right) = \hat{x}_n(\mathbf{w},y,t),\ n = 1,2,\ldots,N.$$

Notice that both primal and dual information is required to recover the input demand functions under the cost-minimizing price-induced TP hypothesis. Thus, specification of the dual cost function $C(\cdot)$ alone is no longer sufficient for recovery of the factor demand functions of firms operating under the influence of price-induced TP.

The following theorem, which is the central result of this section, resolves the ostensible lack of refutable qualitative properties in problem (4) by (i) deriving a symmetric and negative semidefinite matrix that contains its observable curvature properties, and (ii) establishing an upper bound for the rank of the matrix. The proof employs the primal-dual formalism of Silberberg (1974), and is relegated to the appendix for expository purposes.

Theorem 1 (Qualitative Properties). *The curvature properties of the price-taking cost-minimizing model of the firm operating under the influence of price-induced TP defined by Eq.* (4) *et. seq., are summarized by the statement that the* $N \times N$ *matrix* $\mathbf{S}_1(\mathbf{w}, y, t)$, *defined as*

$$\mathbf{S}_1(\mathbf{w}, y, t) \stackrel{\text{def}}{=} C_{\mathbf{ww}} + C_{\mathbf{w}y} f_{\mathbf{w}} + C_y f_{\mathbf{ww}} + f_{\mathbf{w}}{}' C_{y\mathbf{w}} + f_{\mathbf{w}}{}' C_{yy} f_{\mathbf{w}},$$

is negative semidefinite, symmetric, with $\operatorname{rank}\left(\mathbf{S}_1(\boldsymbol{\alpha})\right) \leq N-1$ *for all* $\boldsymbol{\alpha} \in B(\boldsymbol{\alpha}^\circ; \delta)$.

Theorem 1 generalizes the curvature property of the neoclassical cost function in the sense that the curvature property of Theorem 1 contains that of the prototype cost-minimizing model of the firm as a special case. To see this, simply observe that when $f_{\mathbf{w}}(\mathbf{x};\mathbf{w},t) \equiv \mathbf{0}'_N$, problem (4) collapses to the archetype model of the cost-minimizing firm. More precisely, if $f_{\mathbf{w}}(\mathbf{x};\mathbf{w},t) \equiv \mathbf{0}'_N$, then $\mathbf{S}_1(\mathbf{w}, y, t) = C_{\mathbf{ww}}(\mathbf{w}, y, t)$ is a symmetric and negative semidefinite matrix, which is equivalent to the concavity of $C(\cdot)$ in $\mathbf{w}$, the neoclassical result. Theorem 1 and the four remarks preceding it demonstrate that the qualitative properties of the cost-minimizing price-induced TP model differ markedly from those of the archetype cost-minimizing model.

A novel feature of Theorem 1 is the appearance of both the production function and cost function in the comparative statics characterization of problem (4). This property is absent from any prototype model of the firm and it is the distinguishing feature of our model of price-induced TP. It can be viewed as the scaffolding by which one can erect the estimating framework of the price-induced TP hypothesis. In other words, in general, one must always estimate the production function with a dual relation, namely either a cost or profit function, when carrying out an empirical test of the price-induced TP theory presented here. In passing, note that in the appendix we state and prove a second theorem that is equivalent to Theorem 1, but of a different form.

The form of Theorem 2 highlights the comparative statics properties of the cost-minimizing price-induced TP model (4) using the factor demand functions, and provides further evidence of just how this model differs from the archetypal one. Theorem 2 will prove useful in section 6 when we derive the testable implications of the cost-minimizing price-induced TP model for the empirical specification of the production function best supported by the data.

4. California Cotton Ginning

The state of California was the second largest producer of cotton lint and cottonseed in the U.S. in the year 2000, accounting for approximately 15% and 14% of the country's production of lint and cottonseed, respectively. The combined value of these jointly produced goods was over $1 billion in 2000, placing cotton lint and cottonseed as the sixth highest value commodity produced in the state, ahead of such notables as tomatoes and almonds. Moreover, cotton lint was the second leading agricultural export of the state in 2000.

Our cotton ginning data consist of financial and operations information supplied by 22 San Joaquin Valley ginning cooperative firms for the 1980–81 through 1984–85 ginning seasons. We refer the readers to Sexton *et. al.* (1989) for a more extensive discussion of the data and the construction of the basic variables.

Labor expenditures were calculated as the sum of direct and indirect expenditures for full- and part-time employees for each gin. The variable input labor x_L is defined as the annual labor hours worked by the gin's full- and part-time employees, including overtime hours. The wage rate W_L for each gin was computed by dividing labor expenditures by x_L.

Energy expenditures for each gin were measured as the sum of its annual expenditures for electricity, natural gas, and/or propane. British thermal unit (BTU) prices for each fuel were computed from each gin's utility rate schedules. These were then aggregated into a single BTU price, W_E, for each gin using BTU quantity weights for each energy source. The variable input energy x_E was then computed by dividing energy expenditures by W_E.

Because of the gins' lengthy down time each year, typically about nine months, capital is treated as a variable input rather than a fixed or quasi-fixed input. The long down time makes it relatively easy for the gins to make adjustments in the ginning equipment and buildings, the two components of capital in our sample. The data show that such year-to-year adjustments were in fact frequently made. Each component of the capital stock was measured using the perpetual inventory method and straight line depreciation, the latter being derived from the asset's service life. The extended down time between two successive production seasons also gives the opportunity to the gin's managers of choosing and adopting new techniques, thereby providing further support for our inclusion of the relative prices into the production function.

The rental prices of the buildings and ginning equipment were measured by the Christensen and Jorgenson (1969) formula, which accounts for, among other things, the gin's average marginal income tax rate for co-op members, an investment tax credit, and the property tax rate. Expenditures for each component of the capital stock were computed as the product of each component of the capital stock and its corresponding rental rate, and then summed to obtain total capital expenditures. The composite capital rental price, W_K, for each gin was then computed using an expenditure weighted average of the gin's rental prices for buildings and equipment. The composite measure of the capital stock service flow x_K is computed by dividing total capital expenditures by the composite rental price W_K.

As observed by Sexton *et. al.* (1989), the cotton gins take the raw cotton input x_R delivered to them by the grower-members of the co-op as given, and apply the variable inputs capital, labor, and energy, to produce cleaned and baled cotton lint and cotton seed in *fixed proportions*, thereby implying that we may consider them as producing a single composite output y. Moreover, because there is effectively no substitution between the raw cotton input and the three variable inputs, the gins produce the single composite output by way of a quasi-fixed production technology, namely

$$(x_R, x_K, x_L, x_E, w_K, w_L, w_E, t) \mapsto \min\left\{\psi^{-1} x_R, f(x_K, x_L, x_E; w_K, w_L, w_E, t)\right\},$$

where $w_i \stackrel{\text{def}}{=} W_i / P$, $i = K, L, E$, are the relative prices of the three factors of production, P is the composite output price defined subsequently, and ψ is the conversion factor between the raw cotton input x_R and the composite output y. Technical efficiency on the part of the gins implies that $y = \psi^{-1} x_R = f(x_K, x_L, x_E; w_K, w_L, w_E, t)$. Such a technology implies that we are permitted to use the raw cotton input x_R as our composite output variable in the estimation of the production function, i.e., we may estimate the production relationship $x_R = \psi f(x_K, x_L, x_E; w_K, w_L, w_E, t)$. This feature of the ginning process is particularly well suited to the cost minimization problem (4) since the raw cotton input x_R is taken as given by the cotton gins, as noted above.

The price of the composite output y is defined as $P \stackrel{\text{def}}{=} P_c + \phi P_s$, where P_c is the price per 500-pound bale of cotton lint, P_s is the price per ton of cotton seed, and ϕ is the ratio of tons of seeds per 500-pound bale of cotton lint. The ratio ϕ captures the difference, if any, between the picking and stripping methods of removing the raw cotton from the plant. This ratio, however, is not under the control of the gins, as it reflects the choice of stripping technique employed by the cotton member-growers of the co-op. Hence the composite output price is in fact exogenous to the gins, just as problem (4) assumes.

In closing out this section we offer two more pertinent remarks on the data. First, the 1983–84 ginning season was dropped from the sample due to the extraordinary nature of the payment-in-kind program that was in effect that season only and greatly distorted growers' production decisions. Second, of the remaining seasons in the sample, scilicet 1980–81, 1981–82, 1982–83, and 1984–85, 18 firms had the necessary data in the 1980–81 season while 22 did in the remaining three seasons. Taking both of these factors into account reduces our sample size to 84 complete observations.

5. Empirical Evidence on the Functional Form of the Production Function

The empirical implementation of the theory in section 3 suggests a two-stage approach. In the first stage, we determine a parsimonious functional form of the production function that is consistent with the available sample information. In the second stage, we re-estimate the resulting production function jointly with its dual cost function in order to conduct a statistical test of the price-induced TP hypothesis. This amounts to testing the inequality restrictions implied by the negative semidefiniteness property of Theorem 1 and the positive marginal products of the factors of production at the optimum.

We commence, therefore, by gathering empirical evidence on a parsimonious functional form of the production function that fits the sample data. First we specify a flexible and encompassing functional form, namely a time and relative price dependent translog production function. Then we perform statistical tests on it to see if it can be reduced to one of its encompassed specifications, for example, a relative price dependent Cobb-Douglas production function. In addition, we test for the robustness of the final functional form by initially specifying a time and relative price dependent CES production function, and similarly test to see if it can be reduced to a relative price dependent Cobb-Douglas production function. As we will see, all the statistical evidence points to a relative price dependent Cobb-Douglas production function.

It is worthwhile to remark that we undertake a sequence of statistical tests in order to deduce the functional form of the production function that best rationalizes the sample data. As a result, we hold the probability of a type-I error fixed at a relatively low level for each test, videlicet the 0.01 level of confidence. This has the effect of keeping the overall probability of a type-I error reasonably low. Furthermore, we report the *p*-value for the computed statistic. The *p*-value is the tail probability for a two-tailed test of the null hypothesis, and is the exact level of significance of a test statistic. The null hypothesis is therefore rejected if the reported *p*-value is less than the chosen level of significance of 0.01.

To reiterate, we begin by specifying a translog production function that includes capital (x_K), labor (x_L), and energy (x_E), the price of each input divided by the composite output price (w_K, w_L, w_E), i.e., relative input prices, and a time index t, as arguments:

$$\ln y = \alpha_0 + \sum_i \alpha_i \ln x_i + \tfrac{1}{2}\sum_i \sum_j \beta_{ij} \ln x_i \ln x_j + \sum_i \gamma_i \ln w_i + +\tfrac{1}{2}\sum_i \sum_j \delta_{ij} \ln w_i \ln w_j$$
$$+ \sum_i \sum_j \eta_{ij} \ln x_i \ln w_j + \sum_i \phi_i \ln t \ln x_i + \sum_i \varphi_i \ln t \ln w_i + \mu \ln t + \tfrac{1}{2}\theta(\ln t)^2,$$

where $i,j = K,L,E$, symmetry of the second-order coefficients is imposed, that is, $\beta_{ij} = \beta_{ji}$ and $\delta_{ij} = \delta_{ji}$, $i,j = K,L,E$, and the observation indices have been suppressed for notational clarity.

The parameters of the translog production function are estimated using an instrumental variable estimator so as to account for the endogeneity of the factors of production. The instruments include the logarithm of the vector (w_K, w_L, w_E, t) along with its associated combination of quadratic form variables.

First we test the null hypothesis that a prototype Cobb-Douglas production function is consistent with the sample of ginning data, that is, one in which only the factors of production determine output. This null hypothesis amounts to setting all the β_{ij}, γ_i, δ_{ij}, η_{ij}, ϕ_i, φ_i, μ, and θ parameters to zero, for a total of 32 restrictions. The computed Wald χ^2-statistic, with 32 degrees of freedom, is 74.9454 with a *p*-value of 0.00003, leading us to reject this null hypothesis.

We then test the null hypothesis that a time dependent Cobb-Douglas production function is consistent with the sample of ginning data. This null hypothesis amounts to setting all the β_{ij}, γ_i, δ_{ij}, η_{ij}, ϕ_i, φ_i, and θ parameters to zero, for a total of 31 restrictions. The computed Wald χ^2-statistic, with 31 degrees of freedom, is 65.1193 with a *p*-value of 0.00032, leading us to reject this null hypothesis too.

Next we test the null hypothesis that a time and relative price dependent Cobb-Douglas production function is consistent with the sample of ginning data. This null hypothesis amounts to setting all the β_{ij}, δ_{ij}, η_{ij}, ϕ_i, φ_i, and θ parameters to zero, for a total of 28 restrictions. The computed Wald χ^2-statistic, with 28 degrees of freedom, is 47.1052 with a *p*-value of 0.01335. As a result, we cannot reject the null hypothesis in this case. Our conclusion, therefore, is that a time and relative price dependent Cobb-Douglas production function is consistent with the ginning data. Additional statistical evidence supporting this conclusion comes from the fact that the time and relative price dependent Cobb-Douglas production function yielded residuals that passed the goodness of fit test for normality and the Jarque-Bera normality test. Moreover, eight different tests for heteroskedasticity were conducted, and in each instance we could not reject the null hypothesis of homoskedasticity. In passing, note that the *R*-square between observed and predicted for the time and relative price dependent Cobb-Douglas production function is 0.9074.

The same conclusion regarding the functional form of the production function is reached beginning with the time and relative price dependent CES production function

$$y = A\left[\delta_1 x_K^{-\rho} + \delta_2 x_L^{-\rho} + [1-\delta_1-\delta_2]x_E^{-\rho}\right]^{\frac{-\eta}{\rho}}\left[w_K^{\gamma_K} w_L^{\gamma_L} w_E^{\gamma_E} t^{\delta}\right].$$

We estimated this production function using a nonlinear instrumental variable estimator to again account for the endogeneity of the factors of production. The instruments include the vector (w_K, w_L, w_E, t) along with its associated combination of quadratic form variables. The null hypothesis that a time and relative price dependent Cobb-Douglas production function is consistent with the sample of ginning data amounts to testing whether the parameter ρ is zero, for a total of 1 restriction. The computed Wald χ^2-statistic, with 1 degree of freedom, is 0.03972 with a *p*-value of 0.84203. Thus we cannot reject the null hypothesis. In sum, we find ample and consistent statistical evidence in favor of the time and relative price dependent Cobb-Douglas production function, whether we initially postulate a translog or CES production function incorporating time and relative input prices.

Conditional on the conclusion that a time and relative price dependent Cobb-Douglas production function is consistent with the data, we now test to see if time is a significant determinant of the gins' technology. For the null hypothesis that the coefficient on time is zero, the computed asymptotic *t*-statistic, with 76 degrees of freedom, is –1.288 with a *p*-value of 0.198. Hence we cannot reject the null hypothesis that time has no significant impact on the production technology. We may therefore conclude that a relative price dependent Cobb-Douglas production function is consistent with the ginning data. Observe that the *R*-square between observed and predicted for the relative price dependent Cobb-Douglas production function is 0.9135. Conditional on this conclusion, we now test to see if the presence of the relative prices in the Cobb-Douglas production function is statistically significant.

The statistical evidence that relative prices are a significant determinant of the gins' choice of technology is strong. For the null hypothesis that all three coefficients on the relative prices are zero, the computed Wald χ^2-statistic, with 3 degrees of freedom, is 19.7659 with a *p*-

value of 0.00019. We are thus led to reject the null hypothesis that relative input prices are statistically unimportant in influencing the gins' technology, given a relative price dependent Cobb-Douglas production function. In other words, we may conclude that relative input prices are a statistically important determinant of the gins' technology. In sum, therefore, *all* the statistical evidence points to a relative price dependent form of a Cobb-Douglas production function for the cotton gins. This finding concludes the first stage of our test procedure. For ease of reference, we have compiled the various null hypotheses, the *p*-values of each computed test statistic, and the decision pertaining to each of the statistical tests in Table 1.

6. Rigorous Tests of the Price-Induced TP Theory

Given the above first stage conclusion concerning the functional form of the production function, we now turn to the second stage of the empirical work, to wit, the statistical test of the complete set of qualitative properties of the price-induced TP theory. To begin, we first present the general form of the relative price dependent Cobb-Douglas production function deduced from the first stage analysis:

$$y = A x_K^{\alpha_K} x_L^{\alpha_L} x_E^{\alpha_E} w_K^{\gamma_K} w_L^{\gamma_L} w_E^{\gamma_E}. \tag{8}$$

The corresponding dual cost function, found by solving problem (4) using the production function in Eq. (8), is given by

$$C = [\alpha_K + \alpha_L + \alpha_E]\left[A^{-1}\alpha_K^{-\alpha_K}\alpha_L^{-\alpha_L}\alpha_E^{-\alpha_E}\right]^{\frac{1}{\alpha_K+\alpha_L+\alpha_E}} y^{\frac{1}{\alpha_K+\alpha_L+\alpha_E}} w_K^{\frac{\alpha_K-\gamma_K}{\alpha_K+\alpha_L+\alpha_E}} w_L^{\frac{\alpha_L-\gamma_L}{\alpha_K+\alpha_L+\alpha_E}} w_E^{\frac{\alpha_E-\gamma_E}{\alpha_K+\alpha_L+\alpha_E}}. \tag{9}$$

After taking natural logarithms of Eqs. (8) and (9), we estimated the pair of equations as a system jointly using the nonlinear three-stage least squares estimator so as to account for the endogeneity of the factors of production. The instruments include the logarithm of the vector (w_K, w_L, w_E, t) along with its associated combination of quadratic form variables. The results of the estimation are presented in Table 2. To gain confidence that we obtained a global minimum, and not just a local minimum, of the criterion function, we employed several different sets of starting values for the coefficients, three different numerical algorithms, and two different convergence criteria for the coefficients. In every instance we obtained essentially the same set of parameter estimates and standard errors.

It is worthwhile to point out that the joint estimation of the production and cost functions is justified also in the case of a conventional Cobb-Douglas specification. This is so because, in an empirical context, the two functions carry specific information in the form of error components and, therefore, their joint estimation results in efficient parameter estimates.

As a final check on the validity of relative prices in the production function, we tested the null hypothesis that the γ_i, $i = K, L, E$, coefficients are zero using the nonlinear three-stage least squares estimates of Eqs. (8) and (9). The computed Wald χ^2-statistic, with 3 degrees of freedom, is 48.5123 with a p-value of 0.0000. Thus, we again are led to the conclusion that relative prices are statistically important determinants of the gins' production technology, just as we found in section 5 when using single equation estimates of the production function.

In order to carry out a statistical test of the cost-minimizing price-induced TP theory, we must determine if the estimated production and cost functions satisfy the implied inequality restrictions that follow from the negative semidefiniteness of $\mathbf{S}_1(\boldsymbol{\alpha})$ from Theorem 1. To this end, we first compute $\mathbf{S}_1(\boldsymbol{\alpha})$ using Eqs. (8) and (9). After lengthy differential calculus and algebra computations, we arrive at

$$\mathbf{S}_1(\boldsymbol{\alpha}) = \frac{-C}{[\alpha_K + \alpha_L + \alpha_E]^2} \begin{bmatrix} \frac{\alpha_K[\alpha_L + \alpha_E]}{w_K^2} & \frac{-\alpha_K\alpha_L}{w_K w_L} & \frac{-\alpha_K\alpha_E}{w_K w_E} \\ \frac{-\alpha_K\alpha_L}{w_K w_L} & \frac{\alpha_L[\alpha_K + \alpha_E]}{w_L^2} & \frac{-\alpha_L\alpha_E}{w_L w_E} \\ \frac{-\alpha_K\alpha_E}{w_K w_E} & \frac{-\alpha_L\alpha_E}{w_L w_E} & \frac{\alpha_E[\alpha_K + \alpha_L]}{w_E^2} \end{bmatrix}. \tag{10}$$

By Theorem 1.E.11(iv) of Takayama (1985), necessary and sufficient conditions for the negative semidefiniteness of $\mathbf{S}_1(\boldsymbol{\alpha})$ are (i) nonpositivity of all three first-order principal minors, (ii) nonnegativity of all three second-order principal minors, and (iii) a nonpositive determinant. Inspection of Eq. (10) reveals that the nonpositivity of the three first-order principal minors is equivalent to

$$\alpha_K[\alpha_L + \alpha_E] \geq 0,\ \alpha_L[\alpha_K + \alpha_E] \geq 0,\ \alpha_E[\alpha_K + \alpha_L] \geq 0\,, \tag{11}$$

while nonnegativity of the three second-order principal minors is equivalent to

$$\alpha_K \alpha_L \alpha_E[\alpha_K + \alpha_L + \alpha_E] \geq 0\,. \tag{12}$$

Since $|\mathbf{S}_1(\boldsymbol{\alpha})| \equiv 0$, the four inequalities in Eqs. (11) and (12) constitute the testable implications of the negative semidefiniteness of $\mathbf{S}_1(\boldsymbol{\alpha})$ from Theorem 1 given the relative price dependent Cobb-Douglas production function in Eq. (8). Moreover, that $|\mathbf{S}_1(\boldsymbol{\alpha})| \equiv 0$ implies that the rank conclusion of Theorem 1, videlicet $\operatorname{rank}(\mathbf{S}_1(\boldsymbol{\alpha})) \leq N - 1 = 2$, is automatically met by our estimated model. Observe that the inequalities in Eqs. (11) and (12) hold if $\alpha_i \geq 0$ or if $\alpha_i \leq 0$, $i = K, L, E$.

The four nonlinear inequalities in Eqs. (11) and (12), which we have shown to be equivalent to the negative semidefiniteness of $\mathbf{S}_1(\boldsymbol{\alpha})$, can be replaced by three linear inequality restrictions on a subset of the parameters of the Cobb-Douglas production function in Eq. (8). Key to our demonstration is the fact that Eqs. (11) and (12) do not represent the exhaustive set of testable properties of the cost-minimizing price-induced TP model of the firm defined by Eq. (4). To see this, recall that in section 3 we demonstrated that there is another set of qualitative implications associated with problem (4), to wit, the marginal products of the factors of production must necessarily be positive at the optimum. Using Eq. (8), this necessary requirement is that

$$\frac{\partial y}{\partial x_i} = \frac{\alpha_i y}{x_i} > 0\,,\ i = K, L, E \tag{13}$$

at the optimal solution. Since $y > 0$ and $x_i > 0$, $i = K, L, E$, a set of inequalities equivalent to those in Eq. (13) is given by

$$\alpha_K > 0,\ \alpha_L > 0,\ \alpha_E > 0\,. \tag{14}$$

Thus, the inequality restrictions in Eqs. (11), (12), and (14) constitute the complete set of restrictions on the cost-minimizing price-induced TP model of the firm defined by Eq. (4).

Inspection of Eqs. (11), (12), and (14), however, reveals that the inequality restrictions in Eq. (14) imply those in Eqs. (11) and (12), but not the converse. Hence, in carrying out an exhaustive empirical test of the cost-minimizing price-induced TP theory, all we have to test is the three linear inequality restrictions in Eq. (14) pertaining to the coefficients of the factors of production of the relative price dependent Cobb-Douglas production function in Eq. (8). This fact permits a considerable simplification of the empirical test of the cost-minimizing price-induced TP theory since one need only test the three linear inequality restrictions in Eq. (14), rather than the four nonlinear inequality restrictions in Eqs. (11) and (12). It is important to comprehend that this simplification of the test of the cost-minimizing price-induced TP theory is due entirely to the Cobb-Douglas form of the production function.

The inequality restrictions in Eq. (14) might mistakenly lead one to believe that they are equivalent to testing for the quasi-concavity of the production function of the prototype cost-minimizing model, and therefore that the qualitative properties of the price-induced TP theory and the neoclassical version are identical. But as Theorem 1 and the discussion surrounding it in section 3 have clearly demonstrated, the qualitative properties of the cost-minimizing price-induced TP theory differ markedly from those of the neoclassical theory. In addition, one need only refer to Theorem 2 in the appendix to see that the comparative statics properties of the cost-minimizing price-induced TP model are a generalization of, and thus different from, their neoclassical counterparts.

In order to carry out the empirical test of the inequality restrictions in Eq. (14), we employ the Bayesian inequality constrained estimator of Geweke (1986) with 200,000 replications. This estimator yields an estimate of the probability that the inequality restrictions in Eq. (14) are true for a sample of data. With our ginning data we find that the probability that the inequality restrictions in Eq. (14) hold for relative price dependent Cobb-Douglas production function is 1.0. Consequently, the statistical evidence overwhelmingly supports the cost-minimizing price-induced TP theory. In passing, note that the inequality constrained Geweke (1986) estimates are identical to their unconstrained counterparts in Table 2, since the probability that the inequality restrictions in Eq. (14) hold is 1.0.

By referring to the comparative statics results in Paris and Caputo (2001), we may also test whether the data are consistent with the *profit-maximizing* version of the price-induced TP theory. The profit-maximizing version of the theory requires that an additional inequality restriction hold. The additional restriction is a result of the more stringent primal second-order necessary condition of profit maximization, namely, local concavity of the production function. Thus, in addition to the inequality restrictions in Eq. (14), the profit-maximizing price-induced TP theory implies that

$$\alpha_K + \alpha_L + \alpha_E \leq 1. \tag{15}$$

The simple form that this restriction takes is again due the Cobb-Douglas form of the production function. Using the Bayesian inequality constrained estimator of Geweke (1986) with 200,000 replications, we find the probability that the inequality restrictions in Eqs. (14) and (15) hold is 0.0000 for the relative price dependent Cobb-Douglas production function. Thus, in sharp contrast to the cost-minimizing theory of price-induced TP, there is no evidence that the profit-maximizing version of the theory is compatible with the ginning data.

It turns out that these contrasting conclusions are not really surprising if one knows a little bit about the relationship between the cotton gins and the member-growers of the cooperative. In particular, an important feature of the grower and processor relationship is the fact that the member-growers of the cooperative deliver whatever amount of raw cotton that they harvested in a given period to the cooperative gins. The gins, therefore, have no choice but to take the amount of raw cotton delivered by their member-growers as given, that is, as something they have no control over. Because of the aforementioned quasi-fixed proportions production technology $(x_R, x_K, x_L, x_E, w_K, w_L, w_E, t) \mapsto \min\{\psi^{-1} x_R, f(x_K, x_L, x_E; w_K, w_L, w_E, t)\}$, the volume of raw cotton delivered to a gin essentially fixes the amount of cleaned and baled cotton lint it can produce. This, in turn, implies that the cotton gins are fully capable of minimizing the cost of producing a given amount of cleaned and baled cotton lint, but are not able to maximize profits, for the amount of cleaned and baled cotton lint is not under their control, seeing as it is more or less determined by the volume of raw cotton delivered by their member-growers.

We close this section with a discussion of the nature of TP in the California cotton gins studied. As far as factor biases are concerned, because the production function is of the Cobb-Douglas family, the effects of exogenous TP, as given by changes in *t*, or the effects of price-induced TP, as given by changes in the relative factor prices, are neutral, i.e., bias free. We therefore find no evidence of factor bias due to any type of technical change in our admittedly short time-series on the cotton gins. Moreover, given that the time variable is never statistically different from zero in any of the estimated equations, we may further conclude that all the TP in the gins is due to relative input price changes and is therefore of the price-induced variety.

Table 2 shows that a 10% increase in the relative price of capital, ceteris paribus, results in a 3.0% increase in the output of a gin, and a 10% increase in the relative price of labor results in a 6.6% increase in the output of a gin. On the other hand, a 10% increase in the price of energy yields a 3.6% decrease in output. Though the first two results may initially strike one as counterintuitive, our model of price-induced TP is quite general, and as such, it is fully consistent with such a range of empirical features. Moreover, such ostensible counterintuitive results are the fundamental reason why the only complete and rigorous empirical test of the theory is given by the test of its comparative statics properties.

To see that the above price-induced TP calculations are plausible, we simply adopt a different point of view. That is to say, we now examine the effects of a 10% increase in the relative input prices on the cost of production. In addition, we compare the effect on cost of a relative input price increase with and without price-induced TP operating. These calculations are summarized in Table 3, where we note that, in the second column, the cost and production functions were jointly estimated with the parameters γ_i, $i = K, L, E$, set equal to zero to obtain the parameter estimates for the version of the model without price-induced TP. Table 3 shows that for all three relative input prices, production costs rise less when price-induced TP is accounted for than when it is not. For example, a 10% increase in the relative price of capital results in a 3.5% increase in production costs when price-induced TP is assumed to be absent, whereas production costs rise only 1.4% when price-induced TP is accounted for. Similarly, a 10% increase in the relative price of labor results in a 4.3% increase in production costs when price-induced TP is neglected, while production costs fall by 0.2% when price-induced TP is present. On the other hand, a 10% increase in the relative price of energy results in a 2.3% increase in production costs in the absence of price-induced TP, whereas production costs rise by 4.7% when price-induced TP is accounted for. Nonetheless, the first column of Table 3 reveals that price-induced TP has been favorable overall to the cotton gins, in that a simultaneous increase in all three relative prices results in a smaller cost increase than when price-induced TP is assumed to be absent, entirely consistent with what one typically means by technical *progress*.

In passing, we remark that the computation of factor biases, whether price-induced or exogenous, can be straightforwardly carried out within the priced-induced TP theory of the firm exposited here when the production function is not of the Cobb-Douglas variety, as Celikkol and Stefanou (1999) have shown for a generalized Leontief production function.

7. Summary and Conclusions

We formulated an explicit microeconomic model of Hick's (1932, p. 124) conjecture that relative prices influence the magnitude and direction of TP. The resulting formulation permitted us to derive a fundamental set of testable hypotheses that are amenable to empirical implementation in two stages: (i) identification of a statistically acceptable functional form for the production function, and (ii) re-estimation of the resulting production function jointly with its dual cost function in order to conduct a statistical test of the price-induced TP hypothesis. Using a unique time-series/cross-section sample of data on cotton ginning cooperative firms located in the San Joaquin Valley of California, we found strong and consistent statistical evidence in favor of the cost-minimizing version of the price-induced TP theory. In contrast, we found essentially no statistical evidence supporting the profit-maximizing version of the price-induced TP theory. Because the final form of production function is of the Cobb-Douglas family, the effects of exogenous TP, as given by changes in t, or the effects of price-induced TP, as given by changes in the relative factor prices, are neutral. Moreover, since the time variable was never statistically significant in any of the estimated production and cost functions, all the TP in the gins is due to relative input price changes, that is, all TP is of the price-induced variety.

Two directions for future research come to mind when putting the results of the paper and our approach into perspective. First, further empirical tests of the price-induced TP theory are necessary to further confirm or refute the model as a viable explanation of microeconomic level TP. Specifically, a longer time-series of firm level data would be valuable so that one may have observations on the choice of production techniques over an expanded sample period. Second, the intertemporal nature of the technology adoption process begs for a dynamic version of the theory propounded here. We intend to address both of these concerns in our future work.

8. Appendix

Proof of Theorem 1. Because there are N decision variables and one constraint in problem (4), and the classical constraint qualification holds at the optimum due to the fact that $f_{x_n}(\mathbf{x};\mathbf{w},t)>0$, $n=1,2,\ldots,N$, at the optimum, the dimension of the decision space is $N-1$. This implies that any comparative statics matrix derived from problem (4) cannot have a rank greater than $N-1$, since any complete comparative statics characterization of problem (4) cannot contain any more information than that contained in the primal second-order necessary conditions. This fact implies that $\operatorname{rank}\left(\mathbf{S}_1(\boldsymbol{\alpha})\right)\leq N-1$ for all $\boldsymbol{\alpha}\in B(\boldsymbol{\alpha}^\circ;\delta)$.

Given the above rank property, we are permitted to fix $t=t^\circ$ for the purpose of deriving the qualitative properties of problem (4). We therefore focus on the parameters $(\mathbf{w},y)$. Consequently, let $\mathbf{x}^\circ=\hat{\mathbf{x}}(\mathbf{w}^\circ,y^\circ,t^\circ)$ and suppress $t=t^\circ$ from the arguments of the ensuing equations for notational clarity. Then the primal-dual optimization problem associated with problem (4) is defined as

$$0\overset{\text{def}}{=}\min_{\mathbf{w},y}\left\{\mathbf{w}'\mathbf{x}^\circ-C(\mathbf{w},y)\ \text{ s.t. }\ y-f(\mathbf{x}^\circ;\mathbf{w})=0\right\}. \tag{16}$$

Problem (16) may be rewritten as an equivalent unconstrained minimization problem by using the constraint to eliminate y from it, thereby yielding

$$0\overset{\text{def}}{=}\min_{\mathbf{w}}\left\{\mathbf{w}'\mathbf{x}^\circ-C\left(\mathbf{w},f(\mathbf{x}^\circ;\mathbf{w})\right)\right\}. \tag{17}$$

The necessary conditions, which hold at $\mathbf{w}^\circ$ by construction of problem (17), are given by

$$(\mathbf{x}^\circ)' - C_{\mathbf{w}}\big(\mathbf{w}, f(\mathbf{x}^\circ;\mathbf{w})\big) - C_y\big(\mathbf{w}, f(\mathbf{x}^\circ;\mathbf{w})\big) f_{\mathbf{w}}(\mathbf{x}^\circ;\mathbf{w}) = \mathbf{0}_N, \tag{18}$$

$$\begin{aligned}\mathbf{h}'\Big\{&-C_{\mathbf{ww}}\big(\mathbf{w}, f(\mathbf{x}^\circ;\mathbf{w})\big) - C_{\mathbf{w}y}\big(\mathbf{w}, f(\mathbf{x}^\circ;\mathbf{w})\big) f_{\mathbf{w}}(\mathbf{x}^\circ;\mathbf{w}) - C_y\big(\mathbf{w}, f(\mathbf{x}^\circ;\mathbf{w})\big) f_{\mathbf{ww}}(\mathbf{x}^\circ;\mathbf{w}) \\ &- f_{\mathbf{w}}(\mathbf{x}^\circ;\mathbf{w})' C_{y\mathbf{w}}\big(\mathbf{w}, f(\mathbf{x}^\circ;\mathbf{w})\big) - f_{\mathbf{w}}(\mathbf{x}^\circ;\mathbf{w})' C_{yy}\big(\mathbf{w}, f(\mathbf{x}^\circ;\mathbf{w})\big) f_{\mathbf{w}}(\mathbf{x}^\circ;\mathbf{w})\Big\}\mathbf{h} \ge 0, \forall \mathbf{h} \in \Re^N.\end{aligned} \tag{19}$$

Now observe that the choice of $(\mathbf{w}, y)$ used in holding $\hat{\mathbf{x}}(\mathbf{w}, y)$ fixed in the construction of problems (16) and (17) is arbitrary, so long as $(\mathbf{w}, y) \in B\big((\mathbf{w}^\circ, y^\circ);\delta\big)$. Hence the necessary conditions (18) and (19) hold for all $(\mathbf{w}, y) \in B\big((\mathbf{w}^\circ, y^\circ);\delta\big)$. Using this observation in Eq. (19), multiplying it through by minus unity, and then employing the constraint in identity form, namely $y \equiv f\big(\hat{\mathbf{x}}(\mathbf{w},y);\mathbf{w}\big)$ for all $(\mathbf{w}, y) \in B\big((\mathbf{w}^\circ, y^\circ);\delta\big)$, establishes that $\mathbf{S}_1(\mathbf{w}, y)$ is negative semidefinite for all $(\mathbf{w}, y) \in B\big((\mathbf{w}^\circ, y^\circ);\delta\big)$. Symmetry of $\mathbf{S}_1(\mathbf{w}, y)$ for all $(\mathbf{w}, y) \in B\big((\mathbf{w}^\circ, y^\circ);\delta\big)$ follows from the $C^{(2)}$ nature of $f(\cdot)$ and $C(\cdot)$. *Q.E.D.*

Theorem 2 (Comparative Statics). *For the price-taking cost-minimizing model of the firm operating under the influence of price-induced TP defined by Eq.* (4) *et. seq., the* $N \times N$ *matrix*

$$\mathbf{S}_2(\mathbf{w},y,t) \stackrel{\text{def}}{=} \left[\mathbf{I}_N - C_{\mathbf{w}y} f_{\mathbf{x}} - C_y f_{\mathbf{wx}} - f_{\mathbf{w}}{}' C_{yy} f_{\mathbf{x}}\right] \frac{\partial \hat{\mathbf{x}}}{\partial \mathbf{w}},$$

is identical to $\mathbf{S}_1(\mathbf{w}, y,t)$ *for all* $\boldsymbol{\alpha} \in B(\boldsymbol{\alpha}^\circ;\delta)$.

Proof. To prove that $\mathbf{S}_1(\mathbf{w}, y) \equiv \mathbf{S}_2(\mathbf{w},y)$ for all $(\mathbf{w}, y) \in B\big((\mathbf{w}^\circ, y^\circ);\delta\big)$, we again use the fact that Eq. (18) holds for all $(\mathbf{w}, y) \in B\big((\mathbf{w}^\circ, y^\circ);\delta\big)$ to convert it into an identity in $(\mathbf{w}, y)$. Differentiating the resulting identity with respect to $\mathbf{w}$ using the chain rule gives

$$\begin{aligned}&C_{\mathbf{ww}}(\boldsymbol{\alpha}) + C_{\mathbf{w}y}(\boldsymbol{\alpha}) f_{\mathbf{w}}(\hat{\mathbf{x}}(\boldsymbol{\alpha});\mathbf{w},t) \\ &+ C_y(\boldsymbol{\alpha}) f_{\mathbf{ww}}(\hat{\mathbf{x}}(\boldsymbol{\alpha});\mathbf{w},t) + f_{\mathbf{w}}(\hat{\mathbf{x}}(\boldsymbol{\alpha});\mathbf{w},t)' C_{y\mathbf{w}}(\boldsymbol{\alpha}) \\ &\qquad + f_{\mathbf{w}}(\hat{\mathbf{x}}(\boldsymbol{\alpha});\mathbf{w},t)' C_{yy}(\boldsymbol{\alpha}) f_{\mathbf{w}}(\hat{\mathbf{x}}(\boldsymbol{\alpha});\mathbf{w},t) \equiv \Big[\mathbf{I}_N - C_{\mathbf{w}y}(\boldsymbol{\alpha}) f_{\mathbf{x}}(\hat{\mathbf{x}}(\boldsymbol{\alpha});\mathbf{w},t) - C_y(\boldsymbol{\alpha}) f_{\mathbf{wx}}(\hat{\mathbf{x}}(\boldsymbol{\alpha});\mathbf{w},t) \\ &\qquad\qquad - f_{\mathbf{w}}(\hat{\mathbf{x}}(\boldsymbol{\alpha});\mathbf{w},t)' C_{yy}(\boldsymbol{\alpha}) f_{\mathbf{x}}(\hat{\mathbf{x}}(\boldsymbol{\alpha});\mathbf{w},t)\Big] \frac{\partial \hat{\mathbf{x}}(\boldsymbol{\alpha})}{\partial \mathbf{w}}.\end{aligned} \tag{20}$$

Using the identity $y \equiv f(\hat{\mathbf{x}}(\mathbf{w},y);\mathbf{w})$ for all $(\mathbf{w},y) \in B((\mathbf{w}^\circ, y^\circ);\delta)$ in Eq. (20) and recalling the definitions of $\mathbf{S}_1(\mathbf{w},y)$ and $\mathbf{S}_2(\mathbf{w},y)$, it follows from Eq. (20) that $\mathbf{S}_1(\mathbf{w},y) \equiv \mathbf{S}_2(\mathbf{w},y)$ for all $(\mathbf{w},y) \in B((\mathbf{w}^\circ, y^\circ);\delta)$. *Q.E.D.*

Observe that if $f_{\mathbf{w}}(\mathbf{x};\mathbf{w},t) \equiv \mathbf{0}'_N$, then $y \equiv f(\hat{\mathbf{x}}(\boldsymbol{\alpha});\mathbf{w},t)$ implies that $\mathbf{0}'_N \equiv f_{\mathbf{x}}(\hat{\mathbf{x}}(\boldsymbol{\alpha});\mathbf{w},t)\frac{\partial}{\partial \mathbf{w}}\hat{\mathbf{x}}(\boldsymbol{\alpha})$. In turn, this implies that $\mathbf{S}_2(\boldsymbol{\alpha}) = \frac{\partial}{\partial \mathbf{w}}\hat{\mathbf{x}}(\boldsymbol{\alpha})$ is a symmetric and negative semidefinite matrix, the comparative statics statement of the neoclassical cost minimizing model of the firm.

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Table 1
Determination of the Functional Form of the Production Function

Hypotheses	p-value	Decision
H_0: Prototype Cobb-Douglas H_1: Time and Relative Price Dependent Translog	0.00003	Reject H_0
H_0: Time Dependent Cobb-Douglas H_1: Time and Relative Price Dependent Translog	0.00032	Reject H_0
H_0: Time and Relative Price Dependent Cobb-Douglas H_1: Time and Relative Price Dependent Translog	0.01335	Do Not Reject H_0
H_0: Time and Relative Price Dependent Cobb-Douglas H_1: Time and Relative Price Dependent CES	0.84203	Do Not Reject H_0
H_0: Relative Price Dependent Cobb-Douglas H_1: Time and Relative Price Dependent Cobb-Douglas	0.198	Do Not Reject H_0
H_0: Prototype Cobb-Douglas H_1: Relative Price Dependent Cobb-Douglas	0.00019	Reject H_0

Table 2
Joint Estimation of the Relative Price Dependent Cobb-Douglas Production Function and Dual Cost Function

Parameter	Estimated Coefficient	t-ratio
A	22.656	10.928
α_K	0.5054	19.394
α_L	0.6297	12.205
α_E	0.3362	7.6592
γ_K	0.3002	4.3420
γ_L	0.6563	5.1814
γ_E	–0.3569	–2.7378

Table 3
Percent Change in Cost Resulting from a 10% Increase in a Relative Input Price

	With Price-Induced TP	Without Price-Induced TP
Relative Price of Capital	1.40	3.46
Relative Price of Labor	–0.18	4.23
Relative Price of Energy	4.71	2.31